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Noncovariant Local Symmetry in Abelian Gauge Theories

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Abstract

We find noncovariant local symmetries in the Abelian gauge theories. The Nöther charges generating these symmetries are nilpotent as BRST charges, and they impose new constraints on the physical states.

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I. INTRODUCTION

The theory of gauge fields is based on symmetry principles and the hypothesis of locality of fields. The principle of local gauge invariance determines all the forms of the interactions and allows the geometrical description of the interactions [1]. But the quantization of gauge fields leads to difficulties due to the constraints arising from the gauge symmetry. These difficulties of the quantization of constrained systems can be circumvented by the extension of phase space including the anticommuting ghost variables. In this approach, the original gauge symmetry is transformed into the so-called BRST symmetry in the extended phase space [2–4]. Thus, one expects this BRST symmetry must have the same role as the original gauge symmetry. In other words, the BRST symmetry must determine all the forms of the interactions and the algebraic and topological properties of the fields in the quantum theory [5]. This extension of phase (or configuration) space may imply a larger class of symmetry in an extended phase (or configuration) space. Indeed, recent works [6,7] reported the existence of new and larger symmetries in QED.

In this paper, we find the noncovariant local symmetries in Abelian gauge theories, these symmetries are similar to the symmetry of Ref. 6 discovered in QED, but ours are local. Our symmetries are not included in the class of the symmetry of Ref. 7 since they are non-covariant. In addition, we find that the Nöther charges generating these symmetries are nilpotent as BRST charges and that they impose strong constraints on state space.

II. NEW BRST-LIKE SYMMETRY IN ABELIAN GAUGE THEORIES

Consider the BRST and anti-BRST invariant effective QED Lagrangian. (Our BRST treatments are parallel with those of Baulieu's paper [5].)

$$\begin{aligned}\mathcal{L}_{eff} =& -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{2}\bar{s}s(A_\mu^2 + \alpha\bar{c}c) \\ =& -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi + A_\mu\partial^\mu b + \frac{\alpha}{2}b^2 - \partial_\mu\bar{c}\partial^\mu c \end{aligned} \tag{2.1}$$

where $D_\mu = \partial_\mu + ieA_\mu$ is the covariant derivative with the metric $g_{\mu\nu} = (1,-1,-1,-1)$. This effective Lagrangian has rigid symmetries under the following BRST and anti-BRST transformations:

$$\begin{aligned} sA_\mu &= \partial_\mu c, & \bar{s}A_\mu &= \partial_\mu \bar{c}, \\ sc &= 0, & \bar{s}\bar{c} &= 0, \\ s\bar{c} &= b, & \bar{s}c &= -b, \\ s\psi &= -iec\psi, & \bar{s}\psi &= -ie\bar{c}\psi, \\ sb &= 0, & \bar{s}b &= 0. \end{aligned} \tag{2.2}$$

We have introduced an auxiliary field b to achieve off-shell nilpotency of the BRST and the anti-BRST transformations. Under the transformations in Eq. (2.2), the original gauge invariant classical Lagrangian remains invariant while the variation from the gauge-fixing term in the effective Lagrangian in Eq. (2.1), i.e., $A_\mu\partial^\mu b + \frac{\alpha}{2}b^2 \approx -\frac{1}{2\alpha}(\partial_\mu A^\mu)^2$, is canceled by the variation from the ghost term.

The nilpotent Nöther charges generated by the BRST and the anti-BRST symmetries read as

$$Q = \int d^3x\{-(\partial_i F^{io} - \rho)c - b\dot{c}\}, \tag{2.3}$$

$$\bar{Q} = \int d^3x\{-(\partial_i F^{io} - \rho)\bar{c} - b\dot{\bar{c}}\} \tag{2.4}$$

where ρ is a charge density defind by

$$\rho = e\bar{\psi}\gamma_0\psi. \tag{2.5}$$

The transformations in Eq. (2.2) now can be defined as follows:

$$s\mathcal{F}(x) = i[Q, \mathcal{F}(x)\}, \quad \bar{s}\mathcal{F}(x) = i[\bar{Q}, \mathcal{F}(x)\}$$

where the symbol $[\ ,\ \}$ is the graded commutator. In the language of quantum field theory, the BRST charge Q and the anti-BRST $\bar{Q}$ are the generators of the quantum gauge transformation.

In order to recover the probabilistic interpretation of the quantum theory, we must project out all the physical states in positive definite Hilbert space. We achieve this goal by asking for BRST invariance on an extended state space [8]:

$$Q|phys>= 0. \tag{2.6}$$

Now let us consider the BRST-like operators obtained by canonical transformation interchanging the roles of the ghost c and the antighost $\bar{c}$ in Eqs. (2.3) and (2.4) along the following combinations:

$$\bar{Q}_c = \int d^3x\{(\partial_i F^{io} - \rho)\dot{\bar{c}} + b\nabla^2\bar{c}\}, \tag{2.7}$$

$$Q_c = \int d^3x\{(\partial_i F^{io} - \rho)\dot{c} + b\nabla^2 c\}. \tag{2.8}$$

These operators are also nilpotent, i.e., $\bar{Q}_c^2 = Q_c^2 = 0$. In Ref. 9, we presented the geometrical meaning of the operator $\bar{Q}_c$, which can be interpreted as an *adjoint* operator of the BRST operator Q based on Lie algebra cohomology. We find that $Q(\bar{Q})$ and $\bar{Q}_c(Q_c)$ satisfy the supersymmetry-like algebra that closes into an operator Δ,

$$\begin{array}{lll} \{Q, \bar{Q}_c\} = i\Delta, & [\Delta, Q] = 0, & [\Delta, \bar{Q}_c] = 0, \\ \{\bar{Q}, Q_c\} = -i\Delta, & [\Delta, \bar{Q}] = 0, & [\Delta, Q_c] = 0, \\ \{Q, \bar{Q}\} = 0, & \{Q, Q_c\} = 0, & \{\bar{Q}, \bar{Q}_c\} = 0, \end{array} \tag{2.9}$$

where the operator Δ can be expressed in terms of the constraint functions as follows:

$$\Delta = \int d^3x\{(\partial_i F^{io} - \rho)^2 + (\nabla b)^2\}. \tag{2.10}$$

The transformation generated by the operator $\bar{Q}_c$ is defined as $\bar{s}_c\mathcal{F}(x) = i[\bar{Q}_c, \mathcal{F}(x)\}$, and the explicit transformations are

$$\begin{array}{ll} \bar{s}_c A_0 = -\nabla^2\bar{c}, & \bar{s}_c A_i = -\partial_0\partial_i\bar{c}, \\ \bar{s}_c c = (\partial_i F^{io} - \rho), & \bar{s}_c\bar{c} = 0, \\ \bar{s}_c\psi = ie\dot{\bar{c}}\psi, & \bar{s}_c\bar{\psi} = -ie\dot{\bar{c}}\bar{\psi}, \\ \bar{s}_c b = 0. & \end{array} \tag{2.11}$$

Note that the nilpotency of the transformation acting on the ghost field c holds only on-shell.

Remarkably, we can easily show that this noncovariant local transformation is a symmetry of the Lagrangian in Eq. (2.1). Under the transformation in Eq. (2.11), the Lagrangian in Eq. (2.1) changes by a total divergence

$$\bar{s}_c \mathcal{L}_{eff} = \partial_\mu \Lambda^\mu$$

where

$$\Lambda^0 = (\partial_i F^{io} - \rho)\dot{\bar{c}} - b\nabla^2 \bar{c},$$

$$\Lambda^i = (\partial_j F^{jo} - \rho)\partial^i \bar{c} - b\partial^i \dot{\bar{c}} - F^{i0}\Box \bar{c}.$$

It can be shown that this symmetry generates the same kinds of noncovariant Ward identities as those of Ref. 6 except that our identities are multiplied by $\mathbf{q}^2$. Notice the symmetry in Eq. (2.11) still holds true off-shell regardless of the gauge parameter α.

The symmetry generated by the transformation in Eq. (2.11) is realized as the following combination: the gauge-fixing term in the effective Lagrangian in Eq. (2.1), i.e., $A_\mu \partial^\mu b + \frac{\alpha}{2} b^2 \approx -\frac{1}{2\alpha}(\partial_\mu A^\mu)^2$, remains invariant under the transformation and the variation from the ghost term is canceled up to the total derivative by the variation from the original gauge-invariant classical Lagrangian. We would like to draw the reader's attention to different combinations realizing the symmetries between the (anti-)BRST symmetry and the new (anti-)BRST-like symmetry being discussed above. Compared to the BRST symmetry, it is a very interesting property. In a later paper [10], we will discuss its physical meaning and the Ward identity of this symmetry based on the path integral approach, especially focusing on differences from the BRST symmetry.

With the interchange $\bar{c} \to c$, we can also obtain the antiform of the symmetry in Eq. (2.11) generated by the charge Q_c. In the following, we want to focus only on the symmetry generated by the operator $\bar{Q}_c$ because the antiform of the symmetry generated by the charge Q_c can be obtained by the trivial substitution $\bar{c} \to c$.

Since the effective QED Lagrangian, Eq. (2.1), is invariant under the transformation in Eq. (2.11), the physical state $|\Psi>$ must also satisfy the following condition [6,7]:

$$\bar{Q}_c|\Psi>=0. \tag{2.12}$$

If we have more than one supplementary condition, such as Eq. (2.6) and Eq. (2.12), we can deduce further supplementary conditions from them by taking (anti-)commutators of the operators in them, i.e.,

$$\{Q,\bar{Q}_c\}|\Psi>=i\Delta|\Psi>=0 \ \Leftrightarrow \int d^3x\{(\partial_i F^{io}-\rho)^2+(\nabla b)^2\}|\Psi>=0. \tag{2.13}$$

There is no more supplementary condition due to Eq. (2.9). Since the operator Δ consists of non-negative operators and the condition in Eq. (2.13) must be satisfied everywhere, the supplementary condition in Eq. (2.13) can be rewritten as two conditions:

$$(\partial_i F^{io}-\rho)|\Psi>=0 \ \text{ and } \ b|\Psi>=0. \tag{2.14}$$

Evidently these constraints on the physical state automatically satisfy the previous constraints in Eqs. (2.6) and (2.12) while the former conditions can contain additional states which are annihilated by the ghost operators [11]. These two conditions in Eq. (2.14) are the same ones as Dirac's supplementary conditions [12] or the Gupta-Bleuler condition in the gauge-fixing function b [8].

The same kinds of symmetries as in QED also exist in the BRST Landau-Ginzburg Lagrangian and the BRST Chern-Simons Lagrangian. The argument about the Landau-Ginzburg theory is exactly equal to that of QED, except for a trivial modification of the matter part, so we do not repeat it here.

Now we present the noncovariant local symmetries in the BRST Chern-Simons Lagrangian in (2+1)-dimension:

$$\mathcal{L}_{eff}=\frac{\kappa}{4}\epsilon^{\mu\nu\lambda}A_\mu F_{\nu\lambda}+|D_\mu\phi|^2+A_\mu\partial^\mu b+\frac{\alpha}{2}b^2-\partial_\mu\bar{c}\partial^\mu c. \tag{2.15}$$

This effective Lagrangian is invariant under the BRST and the anti-BRST transformations, which is the same form as Eq. (2.2) for QED. The nilpotent Nöther charges generated by the BRST and the anti-BRST symmetries read as

$$Q = \int d^2x\{-(\frac{\kappa}{2}\epsilon^{ij}F_{ij} - \rho)c - b\dot{c}\}, \tag{2.16}$$

$$\bar{Q} = \int d^2x\{-(\frac{\kappa}{2}\epsilon^{ij}F_{ij} - \rho)\bar{c} - b\dot{\bar{c}}\}. \tag{2.17}$$

Then, the nilpotent BRST-like charges $\bar{Q}_c$ and Q_c in the Chern-Simons theory are given by

$$\bar{Q}_c = \int d^2x\{\frac{\kappa}{2}\epsilon^{ij}F_{ij} - \rho)\dot{\bar{c}} + b\nabla^2\bar{c}\}, \tag{2.18}$$

$$Q_c = \int d^2x\{\frac{\kappa}{2}\epsilon^{ij}F_{ij} - \rho)\dot{c} + b\nabla^2 c\}, \tag{2.19}$$

and the transformations generated by the operator $\bar{Q}_c$ are

$$\begin{aligned}
&\bar{s}_c A_0 = -\nabla^2\bar{c}, && \bar{s}_c A_i = -\partial_0\partial_i\bar{c},\\
&\bar{s}_c c = (\tfrac{\kappa}{2}\epsilon^{ij}F_{ij} - \rho), && \bar{s}_c\bar{c} = 0,\\
&\bar{s}_c\phi = ie\dot{\bar{c}}\phi, && \bar{s}_c\phi^* = -ie\dot{\bar{c}}\phi^*,\\
&\bar{s}_c b = 0.
\end{aligned} \tag{2.20}$$

The Lagrangian in Eq. (2.15) changes by a total divergence under the transformations in Eq. (2.20):

$$\bar{s}_c\mathcal{L}_{eff} = \partial_\mu\Lambda^\mu$$

where

$$\Lambda^0 = (\frac{\kappa}{2}\epsilon^{ij}F_{ij} - \rho)\dot{\bar{c}} - b\nabla^2\bar{c} - \frac{\kappa}{2}\epsilon^{ij}A_i\partial_j\dot{\bar{c}},$$

$$\Lambda^i = (\frac{\kappa}{2}\epsilon^{jk}F_{jk} - \rho)\partial^i\bar{c} - b\partial^i\dot{\bar{c}} - \kappa\epsilon^{ij}A_j\Box\bar{c} + \frac{\kappa}{2}A_0\epsilon^{ij}\partial_j\dot{\bar{c}}.$$

The symmetry in Eq. (2.20) still holds true off-shell, regardless of the gauge parameter α. Of course, with the interchange $\bar{c} \to c$, we can also obtain the antiform of the symmetry in Eq. (2.20) generated by the charge Q_c.

Since the Chern-Simons Lagrangian, Eq. (2.15), is invariant under the BRST transformation and the transformations in Eq. (2.20), the physical state $|\Psi>$ of the Chern-Simons theory must satisfy the conditions

$$Q|\Psi >= \bar{Q}_c|\Psi >= 0. \tag{2.21}$$

These conditions induce an additional constraint on the physical state $|\Psi>$:

$$\Delta|\Psi>=0 \;\Leftrightarrow \int d^2x\{(\frac{\kappa}{2}\epsilon^{ij}F_{ij}-\rho)^2+(\nabla b)^2\}|\Psi>=0. \tag{2.22}$$

Consequently, the physical state $|\Psi>$ of the Chern-Simons theory must satisfy the following two conditions, as it does in the case of QED:

$$(\frac{\kappa}{2}\epsilon^{ij}F_{ij}-\rho)|\Psi>=0 \;\text{ and }\; b|\Psi>=0. \tag{2.23}$$

These conditions are the familiar ones with the anyon problem in the Chern-Simons matter systems [13,14]. In Ref. 14, we will discuss the fact that the supplementary conditions in Eq. (2.23) have an important role in the construction of a physical anyon operator in a covariant gauge.

III. SOME REMARKS ON THE BRST-LIKE SYMMETRY

In Sec. II, we presented a noncovariant local BRST-like symmetry in Abelian gauge theories. This symmetry is still compatible with the gauge-fixing condition, i.e., $\bar{s}_c(\partial_\mu A^\mu)=0$, and there is no reason to abandon locality, unlike the claim in Ref. 6.

Notice the transformations in Eqs. (2.11) and (2.20) can be deformed as follows:

$$\begin{aligned}
&\bar{s}_c A_\mu = -\partial_0\partial_\mu\bar{c}, \\
&\bar{s}_c c = -\dot{b}, \qquad\qquad \bar{s}_c\bar{c}=0, \\
&\bar{s}_c b = 0, \\
&\bar{s}_c\psi(\phi) = ie\dot{\bar{c}}\psi(\phi), \quad \bar{s}_c\bar{\psi}(\phi^*) = -ie\dot{\bar{c}}\bar{\psi}(\phi^*).
\end{aligned} \tag{3.1}$$

One can easily verify, under the above transformations, that the effective actions in Eqs. (2.1) and (2.15) also change by a total divergence, like the previous ones. Thus, the above transformations represent another local symmetry existing in the Abelian gauge theories, which is equivalent to the symmetries in Sec. II only on-shell. This symmetry still preserves the locality and corresponds to the localized version of Eq. (3) of Ref. 7.

Based on the existence of the local symmetry above, one may also expect a nonlocal symmetry corresponding to the local one as long as we demand good boundary conditions

on the fields [6] (see also Ref. 7, especially Eq. (4)). In fact, the nonlocal symmetries in Refs. 7 and 8 are examples showing the existence under the special requirement with respect to the operators generating the nonlocality.

In a recent work [15], we have shown that there exists a straightforward way to isolate the physical Hilbert space with a positive-definite norm through the BRST cohomology. In that paper, we introduced the "co-BRST" operator under a positive-definite inner product and obtained the Hodge decomposition theorem in the state space of BRST quantization. The co-BRST operator in Ref. 15 is quite different from the operator $\bar{Q}_c$ discussed in this paper. The BRST-like charges, Eqs. (2.7) and (2.8), can be used to refine the characterization of the physical states as in Eq. (2.12) and Eq. (2.14), but they cannot be applied to the problem to directly isolate the physical state with a positive-definite norm in the sense of Ref. 15.

For the case of free QED, we have written down the explicit form of the co-BRST operator using the mode expansion of the field operators and have shown it to be a conserved operator. If the BRST Hamiltonian in the system under consideration is Hermitian, the co-BRST operator introduced in our paper is, in general, a conserved charge that commutes with the BRST Hamiltonian. Thus, there may exist a symmetry transformation generated by the co-BRST operator. However, it seems it is very difficult to express the co-BRST transformation in an explicit form in configuration space, even in free theory and it will be different from the symmetry discussed in this paper and in Refs. 6 and 7. For that reason, it will be a very interesting problem to invesigate a larger class of symmetry including the co-BRST symmetry in gauge theories.

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